Jurnal Sains dan Matematika Vol: 1, No 1, 2025
ISSN: 3026-XXXX

Theoretical and Practical Perspectives on Mathematical Modelling of Water Contaminants in Klang River: Analysis and Insights into Pollution Dynamics

Emmanuel Sabastine[1], Saratha Sathasivam[2*], Lee Xin Yong[3]
[123]School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM Pulau Pinang, Malaysia.

DOI: https://doi.org/10.xxxxx/xxxxx
*Correspondence: Saratha Sathasivam
Email: saratha@usm.my
Received: March 18, 2025
Accepted: March 20, 2025
Published: March 21, 2025

Abstract: Understanding pollutant transport dynamics in fluvial systems and formulating effective environmental management strategies necessitate robust water quality modelling. This study develops a mathematical framework to simulate the transport and transformation of key water pollutants—Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), and Total Dissolved Solids (TDS)—within the Klang River, Selangor, Malaysia. The model incorporates multiple pollution sources and temporal variations in contaminant inputs to represent real-world conditions. The governing system of differential equations, which encapsulates pollutant advection, diffusion, degradation, and external influxes, is numerically solved using MATLAB. Three principal analyses are performed: (i) Sensitivity Analysis to determine critical parameters influencing pollutant concentration dynamics; (ii) Scenario Analysis to assess the impact of varying pollution levels, including industrial effluents and seasonal fluctuations; and (iii) Stability Analysis to evaluate long-term pollutant behaviour and the river's intrinsic self-purification capacity. Simulation results indicate that temporal variations in pollutant loading and degradation rates significantly influence water quality trends. Elevated industrial discharges substantially increase BOD and COD concentrations, heightening the risk of hypoxic conditions and ecological degradation. Additionally, model predictions suggest that under specific conditions, pollutant concentrations may reach equilibrium, signifying a dynamic balance between natural attenuation processes and continuous external inputs. The study's findings offer valuable insights into aquatic pollution control and sustainable water resource management. They provide a scientific basis for policy recommendations concerning wastewater treatment, industrial discharge regulations, and riverine ecosystem preservation. Future research will incorporate empirical water quality datasets to further refine model validation and enhance predictive accuracy.

Keywords: Environmental Management, pollution Sources, Pollution Transport, Numerical Modelling, Matlab Simulation

Introduction

Water quality is a critical factor in maintaining ecological balance and ensuring public health. The Klang River in Selangor, Malaysia, is a vital water resource for the surrounding communities, supporting both domestic and industrial activities. However, rapid urbanization and industrialization have significantly impacted the river's water quality, leading to increased pollution levels. Monitoring and predicting water quality parameters such as Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD, and Total Dissolved Solids (TDS)

The Streeter-Phelps equations, which were first given as a weakly coupled set of partial differential equations, describe the transport of pollutants and dissolved oxygen in a river over time and space. The formulation considers biological oxygen demand (BOD), which evaluates how much oxygen is required to break down organic pollutants, as well as dissolved oxygen concentration. Hypotheses include advective substrate transfer with water flow, simultaneous BOD decomposition and oxygen consumption, and atmospheric river reoxygenation [3]. The equations have been extended to include diffusion and dispersion effects. Because they behave similarly, dispersion and diffusion are called hydrodynamic dispersion [12]. Many natural rivers rely on advection to transfer substrates, which neglects dispersion [2]. However, rivers with low flow velocity may rely on dispersion for pollution transport [2]. In 1925, Streeter and Phelps developed the equations with linear decay kinetics for a US Government report on the Ohio River [13]. Many improvements have been made to the decay dynamics to better represent observed data [3]. [5] reported a second-order BOD decay, [6] a Michaelis-Menten expression, and [7] an Nth-order decay [3, 6, 7]. These changes are nonlinear and can be made without fully coupling the equations. Because BOD decomposition is independent of dissolved oxygen, these changes cannot prevent a negative oxygen concentration. To better comprehend the link between BOD degradation and DO concentration, [8] and [7, 9] propose a linear dependence [7, 9, 10]. Create a BOD to decrease reliance on the model, which no longer forecasts negative oxygen concentration [3]. Another alteration is the nonlinearity of Monod coupling [11. Nonlinear modification influences BOD decay concerning DO concentration, with a linear fall at high DO concentration and proportionate degradation at low DO concentration [7]. Several studies have explored the various factors that influence water quality, highlighting key environmental, chemical, and anthropogenic contributors. These factors include pollution from industrial and agricultural activities, variations in temperature and climate, sedimentation, nutrient loading, and microbial contamination. Additionally, hydrological changes such as urbanization, dam construction, and deforestation significantly impact water quality by altering natural flow patterns and introducing pollutants. The studies referenced in [22-31] provide detailed insights into these influences, examining their effects on aquatic ecosystems, human health, and water management strategies.

The Klang River is a vital waterway in Selangor, Malaysia, it has become one of the most polluted rivers in the country due to various anthropogenic activities. Studies have revealed that approximately 80.1% of the river's pollution originated from sewage treatment plants, while other contributors include food outlets and restaurants (3.9%), industrial waste (3.4%), and miscellaneous sources such as workshops, residential areas, and wet markets (12.6%). These pollutants have a detrimental impact on the river's ecosystem and water quality, posing significant challenges to environmental sustainability and public health.

Several factors contribute to the increasing pollution in the Klang River. As it flows through the Klang Valley, a highly urbanised and densely populated region with over four million residents, the river is subjected to untreated human waste from informal settlements along its banks and certain businesses without proper septic tanks or sewage treatment systems. Soil erosion from nearby mountains also introduces sediment into the river, further degrading its quality. Additionally, heavy urban development has narrowed sections of the river, reduced its flow capacity and transformed it into a storm drain. These factors not only exacerbate pollution but also contribute to flash floods in Kuala Lumpur, particularly during periods of heavy rainfall.

Mathematical modelling plays a significant role in understanding pollution transmission and predicting water quality trends in river systems [16, 17, 18, 19, 20]. It provides a quantitative framework for measuring the effects of diverse pollution sources and environmental conditions, enabling decision-makers to establish successful pollution control measures. Traditional models, such as the Streeter-Phelps equation, have been widely utilised to represent the oxygen dynamics in rivers by considering biochemical oxygen demand (BOD) decay and dissolved oxygen (DO) depletion due to organic matter decomposition [2,7]. However, these models are largely developed for single-source pollution inputs and assume steady-state conditions, rendering them insufficient for reflecting the complexities of current river systems influenced by various pollution sources and erratic inflows.

Recent studies underscore the need for more complete models combining many interacting elements, including industrial discharge, seasonal variations, and hydrodynamic changes [4, 14]. This study expands standard pollutant transport models by integrating numerous pollution sources and time-dependent inflow changes, boosting accuracy in reproducing real-world pollutant dynamics. By allowing for regional and temporal variability, the proposed model gives a more accurate portrayal of water quality changes, enabling a better understanding of pollutant dispersion and long-term environmental repercussions. The numerical implementation in MATLAB enables the simulation of diverse pollution scenarios, aiding in predictive analysis and decision-making for sustainable river management [15].

The primary objective of this research is to develop a comprehensive water quality model that accurately simulates pollutant transport and dynamics in the Klang River, Selangor, Malaysia, with a focus on Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), and Total Dissolved Solids (TDS). This study extends existing modeling methodologies by integrating various pollution sources and time-dependent influx rates, hence boosting the model's ability to capture real-world complexities. The precise objectives are as follows:

1. Develop an Enhanced Water Quality Model - Formulate a mathematical model that accounts for pollution movement, degradation, and hydrodynamic variability in river systems. Extend current models (e.g., Streeter-Phelps) to include numerous pollution sources (industrial discharge, domestic wastewater, and agricultural runoff) and time-dependent inflow changes and to implement the model in MATLAB for numerical simulation and analysis.
2. Simulate the Pollutant Dynamics of BOD, COD, and TDS: Analyze how contaminants evolve under diverse environmental settings, examine the impact of pollutant degradation rates, hydrological changes, and anthropogenic activities on water quality and generate spatiotemporal concentration profiles to illustrate pollution dispersal in the river.
3. Perform Sensitivity Analysis to Identify Key Influencing Factors: Evaluate how differences in pollution input rates, degradation coefficients, and flow conditions affect pollutant concentrations and identify the most sensitive metrics influencing water quality to support targeted pollution mitigation efforts.
4. Conduct Scenario Analysis to Assess Different Pollution Conditions - Model several pollution scenarios, including: Low, moderate, and high industrial discharge rates, seasonal changes in river flow (e.g., monsoon vs. dry season), implementation of wastewater treatment strategies and compare water quality under different situations to identify viable mitigation techniques.
5. Analyze Long-Term Stability of Pollutant Concentrations: Determine if pollutant levels stable, oscillate, or rise over time, investigate the equilibrium state of the river system and analyse if pollution management techniques may lead to a stable, self-sustaining water

quality condition and explore the potential for irreversible degeneration in the absence of appropriate therapies.

This research presents scientific insights and practical recommendations for environmental authorities, politicians, and researchers. By integrating advanced numerical simulations with real-world pollution situations, the research aids in developing evidence-based strategies for water pollution control. Enhancing wastewater treatment strategies to promote river health and supporting sustainable water resource management in urban river systems.

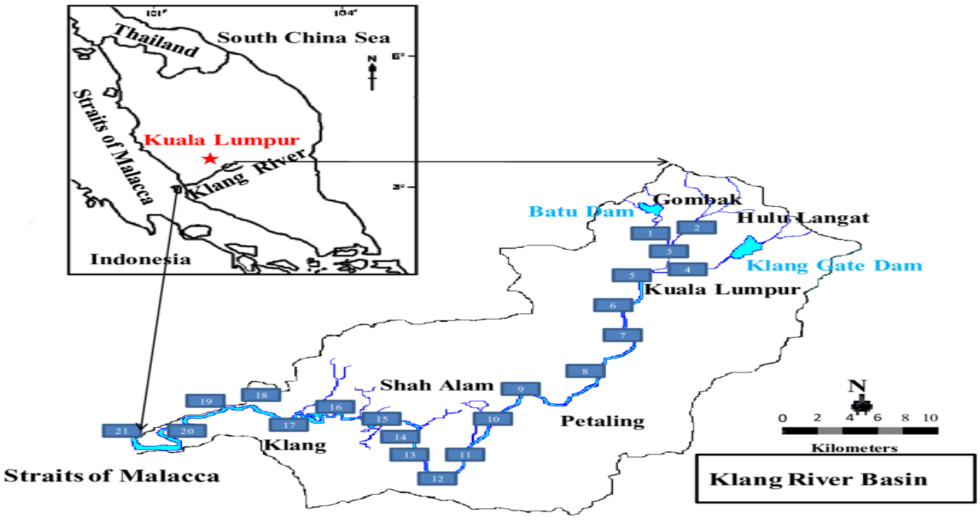

Figure 1: The Map of Klang River Basin, Discussed in [35]

The Klang River Basin is a vital water system in Malaysia, flowing through major urban centres such as Kuala Lumpur, Petaling, Shah Alam, and Klang, before emptying into the Straits of Malacca. The river originates from the highlands near Hulu Langat and Gombak and is regulated by key reservoirs such as Batu Dam and Klang Gate Dam. The river system is crucial for water supply, flood management, and ecological balance but faces pollution challenges due to urbanization and industrial activities. The monitoring stations marked along the river indicate points of interest for water quality assessment. The map highlights the connectivity of the river system and its importance in supporting urban and environmental sustainability [35].

1.0 The Modified Water Quality Model

The time-dependent behaviour of BOD, COD, and TDS can be described by extending the original equation:

$$\frac{dC_i}{dt} = \frac{Q_{in}C_{i,in} - Q_{out}C_{out,in}}{V} + S_i - \beta_i C_i \quad (1)$$

where:

Multiple Pollution Sources: If there are multiple discharges, sum their effects

$$S_i = \sum_{j=1}^{N} \frac{Q_j C_{ij}}{V} \quad (2)$$

Time-Dependent Inflow: Seasonal variations can be introduced, e.g.

$$Q_{in}(t) = Q_{mean} + Asin(\alpha t) \quad (3)$$

- C_i = Concentration of **BOD, COD, or TDS** at time t (mg/m^3).
- Q_{in}, Q_{out} = Inflow and outflow rates (m^3/day).
- $C_{i,in}$ = Inflow of pollutant concentration (mg/m^3).
- S_i = Source/sink term (e.g., pollutant discharge, sediment interactions).
- β_i = Decay/removal rate (1/day).

BOD Decay: BOD is consumed by microbial activity over time, modeled as

$$\frac{d_{BOD}}{dt} = \frac{Q_{in}BOD_{in} - Q_{out}BOD}{V} - K_d BOD \quad (4)$$

where K_d is the **BOD degradation rate** (1/day)

COD Decay: COD has a fraction that degrades (biodegradable) and a fraction that remains unchanged (non-biodegradable)

$$\frac{d_{COD}}{dt} = \frac{Q_{in}COD_{in} - Q_{out}COD}{V} - K_c COD \quad (5)$$

where K_c is the **COD degradation rate** (1/day)

TDS Transport: TDS typically behaves as a **conservative** pollutant (i.e., no decay), and its equation reduces to:

$$\frac{d_{TDS}}{dt} = \frac{Q_{in}TDS_{in} - Q_{out}TDS}{V} + S_{TDS} \quad (6)$$

where S_{TDS} accounts for **sediment resuspension or additional sources**.

2.0 Data Availability

In this project, the data of the volume of the rivers (V), inflow and outflow rates of water (Q_{in} and Q_{out}), the pollutants concentration in the river (C_{in}), and the initial pollutant concentration $C(0)$ can be obtained from credible sources, including scientific articles, which are [33, 34]. These references provide a solid foundation for the parameters and assumptions used in the mode.

The following data were used in the project:

(a) Volume of the River (V)

Given that the drains a basin is about 1,288 km^2 and the average elevation of the river is around $236ft$. So, we need to change the units of the data to make them the same unit.

$1{,}288 km^2 = 1{,}288 \times 1{,}000 \times 1\ 000 = 1.288 \times 10^9\ m^2$
$236\ ft = 236 \times 0.3048 = 71.933\ m$

Thus,
$V = (1.288 \times 10^9 \times 71.933) m^3$

(b) Inflow and Outflow Rates of Water (Q_{in} and Q_{out})

From the articles, the inflow rate and outflow rate of water are the same, which is 50 m^3/s. Now, we want to change it into the units m^3/day
$Q_{in} = Q_{out} = 50 m^3/s = (50 \times 60 \times 60 \times 24)\ m^3/day$

(c) Pollutant Concentrations in Inflow (C_{in})

From the articles, the C_{in} of BOD is $20\ mg/L$, C_{in} of COD is $29\ mg/L$, and C_{in} of TDS is $188.3\ mg/L$. Again, we want to change the units into mg/m^3. Thus,
BOD: $C_{in} = 20\ mg/L = (20 \times 1000)\ mg/m^3$
COD: $C_{in} = 29\ mg/L = (29 \times 1000)\ mg/m^3$
TDS: $C_{in} = 188.3\ mg/L = (188.3 \times 1000)\ mg/m^3$

(d) Initial Pollutant Concentrations $\left(C(0)\right)$

From the articles, we have the train set data, which is from 2019 and the test set data, which is from 2023.
Train Set:
BOD: $C(0) = 4\ mg/L = (4 \times 1000)\ mg/m^3$
COD: $C(0) = 16\ mg/L = (16 \times 1000)\ mg/m^3$
TDS: $C(0) = 100\ mg/L = (100 \times 1000)\ mg/m^3$

Test Set:
BOD: $C(0) = 9.2\ mg/L = (9.2 \times 1000)\ mg/m^3$
COD: $C(0) = 19\ mg/L = (19 \times 1000)\ mg/m^3$
TDS: $C(0) = 119.3\ mg/L = (119.3 \times 1000)\ mg/m^3$

4.4 Growth Rate Constant of BOD, COD, and TDS Concentration.
Given that the IVP is

$$\frac{dC(t)}{dt} = \frac{Q_{in} \cdot C_{in} - Q_{out} \cdot C(t)}{V} + k \cdot C(t)$$

To solve to get, the maximum growth rate constant of k, we let $\frac{dC(t)}{dt} = 0$, we can have the growth rate constant of BOD, COD, and TDS concentration of the train set.

i. BOD Concentration

$$\frac{dC(t)}{dt} = \frac{Q_{in} \cdot C_{in}^{BOD} - Q_{out} \cdot C(t)}{V} + k_B \cdot C(t) = 0, \quad C(0) = (4 \times 1000)\ mg/m^3$$

Hence, by substituting $C(0)$ into the IVP, we get

$$k_B = \frac{Q_{out} \cdot C(0) - Q_{in} \cdot C_{in}^{BOD}}{V \cdot C(0)}$$

Estimating some of the parameters using the data from [33, 34], we have

$$k_B = \frac{(50\times60\times60\times24)[(4\times1000)-(20\times1000)]}{(1.288\times10^9\times71.933)\cdot(4\times1000)}$$

$$k_B = -0.000186629$$

ii. COD Concentration

$$\frac{dC(t)}{dt} = \frac{Q_{in}\cdot C_{in}^{COD} - Q_{out}\cdot C(t)}{V} + k_C \cdot C(t) = 0, \quad C(0) = (16 \times 1000)\ mg/m^3$$

Hence, by substituting $C(0)$ into the IVP, we get

$$k_C = \frac{Q_{out}\cdot C(0) - Q_{in}\cdot C_{in}^{COD}}{V\cdot C(0)}$$

$$k_C = \frac{(50\times60\times60\times24)[(16\times1000)-(29\times1000)]}{(1.288\times10^9\times71.933)\cdot(16\times1000)}$$

$$k_C = -0.000037909$$

iii. TDS Concentration

$$\frac{dC(t)}{dt} = \frac{Q_{in}\cdot C_{in}^{TDS} - Q_{out}\cdot C(t)}{V} + k_S \cdot C(t) = 0, \quad C(0) = (100 \times 1000)\ mg/m^3$$

Hence, by substituting $C(0)$ into the IVP, we get

$$k_S = \frac{Q_{out}\cdot C(0) - Q_{in}\cdot C_{in}^{TDS}}{V\cdot C(0)}$$

$$k_S = \frac{(50\times60\times60\times24)[(100\times1000)-(188.3\times1000)]}{(1.288\times10^9\times71.933)\cdot(100\times1000)}$$

$$k_S = -0.000041198$$

Since the constant k is the growth rate of pollutants; thus we assume they are positive, which are $k_B = 0.000186629$, $k_C = 0.000037909$, and $k_S = 0.000041198$.
For the test set, which is data in 2023, we have

(a) BOD Concentration

$$\frac{dC(t)}{dt} = \frac{Q_{in}\cdot C_{in}^{BOD} - Q_{out}\cdot C(t)}{V} + k_B \cdot C(t) = 0, \quad C(0) = (9.2 \times 1000)\ mg/m^3$$

Hence, by substituting $C(0)$ into the IVP, we get

$$k_B = \frac{Q_{out} \cdot C(0) - Q_{in} \cdot C_{in}^{BOD}}{V \cdot C(0)}$$

$$k_B = \frac{(50 \times 60 \times 60 \times 24)[(9.2 \times 1000) - (20 \times 1000)]}{(1.288 \times 10^9 \times 71.933) \cdot (9.2 \times 1000)}$$

$$k_B = -0.000054736$$

(b) COD Concentration

$$\frac{dC(t)}{dt} = \frac{Q_{in} \cdot C_{in}^{COD} - Q_{out} \cdot C(t)}{V} + k_C \cdot C(t) = 0, \quad C(0) = (19 \times 1000)\ mg/m^3$$

Hence, by substituting $C(0)$ into the IVP, we get

$$k_C = \frac{Q_{out} \cdot C(0) - Q_{in} \cdot C_{in}^{COD}}{V \cdot C(0)}$$

By using the data of parameters from part 4.3, we have

$$k_C = \frac{(50 \times 60 \times 60 \times 24)[(19 \times 1000) - (29 \times 1000)]}{(1.288 \times 10^9 \times 71.933) \cdot (19 \times 1000)}$$

$$k_C = -0.00002454$$

(c) TDS Concentration

$$\frac{dC(t)}{dt} = \frac{Q_{in} \cdot C_{in}^{TDS} - Q_{out} \cdot C(t)}{V} + k_S \cdot C(t) = 0, \quad C(0) = (119.3 \times 1000)\ mg/m^3$$

Hence, by substituting $C(0)$ into the IVP, we get

$$k_S = \frac{Q_{out} \cdot C(0) - Q_{in} \cdot C_{in}^{TDS}}{V \cdot C(0)}$$

$$k_S = \frac{(50 \times 60 \times 60 \times 24)[(119.3 \times 1000) - (188.3 \times 1000)]}{(1.288 \times 10^9 \times 71.933) \cdot (119.3 \times 1000)}$$

$$k_S = -0.000026967$$

Since the constant k is the growth rate of pollutants; thus, we assume they are positive, which are $k_B = 0.000054736$, $k_C = 0.00002454$, and $k_S = 0.000026967$.

Algorithm for Water Quality Simulation (BOD, COD, TDS)

Step	Process Description
1. Initialize Parameters	- Set the simulation time (0 to 400 days) with time step dt. - Define initial concentrations for BOD, COD, and TDS. - Assign decay rates: $K_d BOD$, $K_d COD$ and accumulation rate $K_d TDS$ - Consider different initial pollution loads.
2. Compute Concentration Changes	- Update concentrations at each time step using the equations: **BOD decay:** $BOD(t+dt) = BOD(t)\ e$^{−k_{BOD} dt}. **COD decay:** $COD(t+dt) = COD(t)\ e$^{−k_{COD} dt}. **TDS accumulation:** $TDS(t+dt) = TDS(t) + k$_{TDS}dt
3. Store and Plot Results	- Record computed values for BOD, COD, and TDS over time. - Plot concentration curves to observe trends and stabilization points.
4. Interpretation and Analysis	- Analyze how decay rates affect pollutant breakdown. - Evaluate the impact of pollution loads on concentration dynamics. - Examine the continuous accumulation of TDS and its long-term effects.

5. Conclusion & Recommendations	- Assess the efficiency of self-purification for BOD and COD. - Identify the risks posed by increasing TDS levels. - Suggest pollution control strategies such as wastewater treatment, industrial discharge regulation, and solid accumulation management.

This tabular style offers a representation of the method that is both straightforward and well-structured, making it simple to comprehend and put into practice.

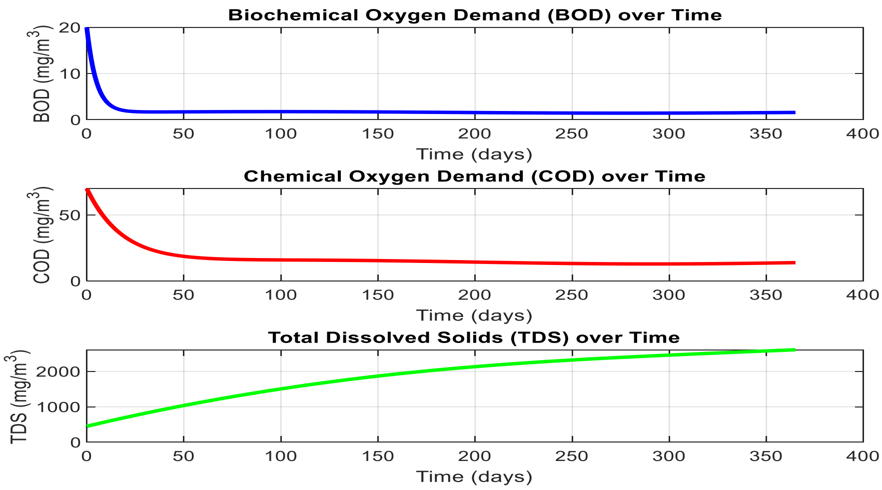

Figure 2: The Graphical Representation of BOD, COD and TDS Demand over Time

This figure 2, consists of **three subplots** showing the variation of **Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), and Total Dissolved Solids (TDS)** over **400 days**. Each parameter is plotted against time to observe long-term trends in pollutant behavior.

First Plot: BOD (mg/m^3)

- Behavior: The blue curve starts with a high BOD concentration (~20 mg/m^3) and rapidly decreases within the first 50 days, eventually stabilizing at a low value.
- Interpretation:
  - BOD represents the amount of organic matter that needs oxygen to decompose.
  - The rapid decline suggests efficient microbial degradation of organic pollutants.
  - The stabilization at a near-zero value indicates that organic pollution has mostly been broken down.

Second Plot: COD (mg/m^3)

- Behavior: The red curve starts high (~60 mg/m^3), decreases over time, but stabilizes at a nonzero value.
- Interpretation:

- COD measures the total oxygen demand, including biodegradable and non-biodegradable pollutants.
- The decrease suggests that some organic pollutants are decomposed.
- However, the stabilization at a higher level than BOD suggests the presence of non-biodegradable pollutants like heavy metals and synthetic chemicals.

Third Plot: TDS (mg/m^3)

- Behavior: The green curve shows a continuous increase in TDS over time, reaching a high concentration.
- Interpretation:
  - TDS includes dissolved ions such as salts, minerals, and metals.
  - The increasing trend suggests continuous accumulation of dissolved solids, possibly due to external pollutant inputs, evaporation, or lack of natural dilution.
  - Unlike BOD and COD, which decrease due to degradation, TDS tends to persist and accumulate over time.

Observation: BOD reduction suggests effective biodegradation. **COD stabilizing at a nonzero level** indicates persistent chemical pollutants. **TDS accumulation** shows increasing salinity or contamination from external sources.

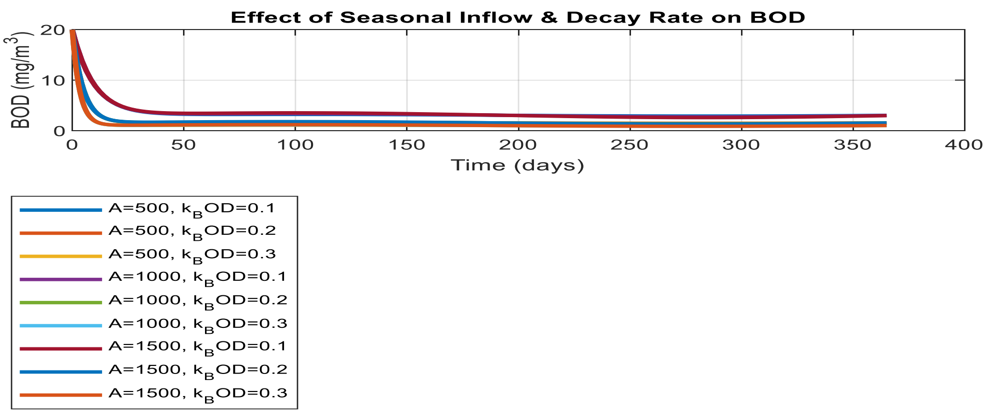

Figure 3: The Graphical Representation Showing the Effect of Seasonal Inflow and Decay Rate

The graph in figure 3 illustrates the variation of Biochemical Oxygen Demand (BOD) over 400 days for different initial pollution loads (A) and decay rate constants $K_d BOD$. Initially, BOD levels drop sharply before stabilizing at lower concentrations. Higher $K_d BOD$ values lead to faster decomposition of organic matter, improving water quality more quickly. Conversely, larger initial pollution loads (A) result in higher starting BOD levels but follow a similar decay trend. After approximately 100 days, the majority of BOD reduction is achieved, indicating that the natural breakdown process is effective over time.

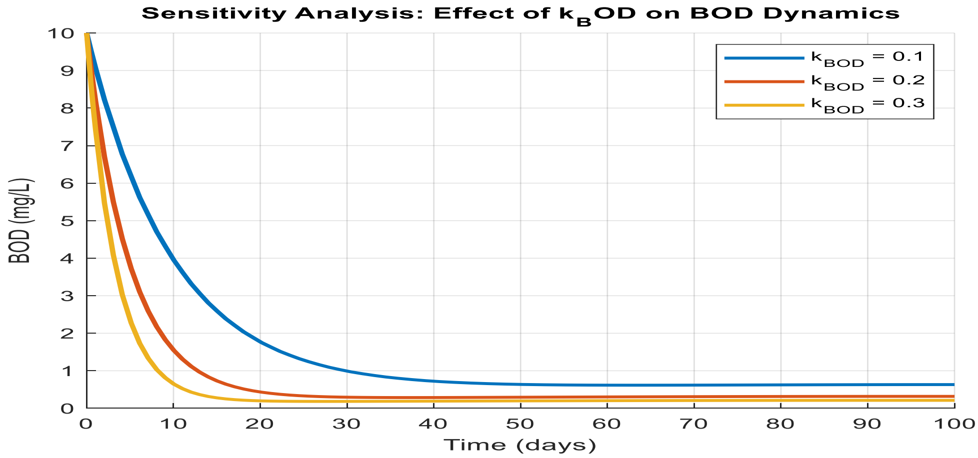

Figure 4: The graphs of Sensitivity analysis showing the Effect of BOD

Interpretation of Each Curve in Figure 3

- Higher decay rates $K_d BOD$ lead to faster BOD reduction.
  - The yellow curve $K_d BOD = 0.3$ shows the fastest decay, reaching nearly zero in less than 10 days.
  - The red curve $K_d BOD = 0.2$ takes about 20 days to approach zero.
  - The blue curve $K_d BOD = 0.1$ has the slowest decay, requiring over 40 days for a significant reduction.
- All curves start at the same initial BOD concentration (~10 mg/L).
  - This represents an initial pollution load (e.g., wastewater discharge).
  - The faster the decay rate, the quicker the river recovers from organic pollution.

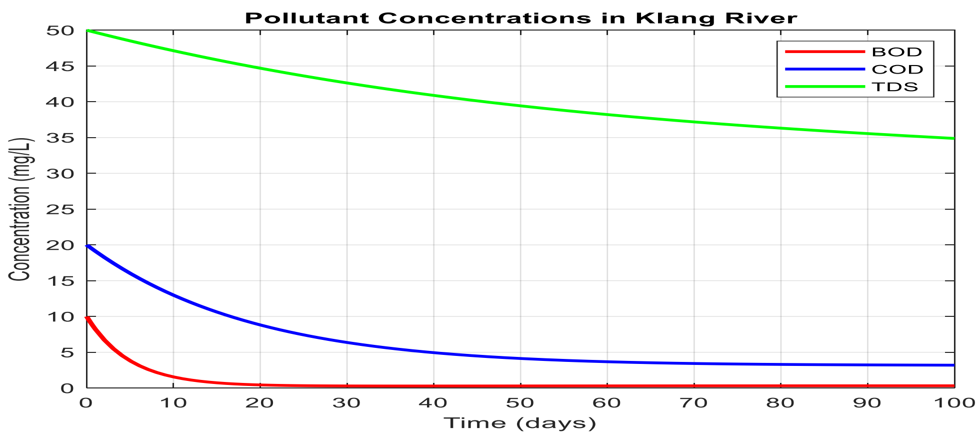

Figure 5: The graphs of Pollutant Concentration

Fiure 5: This graph represents the concentration of three key pollutants—Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), and Total Dissolved Solids (TDS)—in Klang River over 100 days. The simulation captures how these pollutants decrease over time due to natural decay and river dynamics.

Interpretation of Each Curve

BOD (Red Curve):

- Starts at 10 mg/L and rapidly decreases within the first 20 days before stabilizing close to zero.
- This indicates that BOD depletes quickly, likely due to microbial degradation.
- The decay rate of BOD $K_d BOD = 0.2/day$ is relatively high, leading to faster removal.

COD (Blue Curve):

- Starts at 20 mg/L and declines gradually over 100 days.
- Unlike BOD, COD represents both biodegradable and non-biodegradable organic matter.
- The slower decay rate ($k\_COD = 0.05/day$) means some persistent pollutants remain in the river.

TDS (Green Curve):

- Starts at 50 mg/L and decreases very slowly over time.
- TDS represents dissolved salts, minerals, and inorganic matter, which are less affected by natural degradation.
- The decay rate is the lowest ($k\_TDS = 0.01/day$), meaning it remains present in the river much longer.

Key Observations: BOD is removed first. → Suggests good microbial activity in the river. COD takes longer to degrade. → Some pollutants require more time or external intervention.
TDS is persistent. → Requires physical processes (dilution, sedimentation) for significant reduction.

Environmental Implications

- The river recovers from organic pollution (BOD, COD), but long-term pollution (TDS) remains a concern.
- High initial BOD levels indicate pollution sources like wastewater discharge, which degrade quickly.
- Persistent pollutants like TDS require additional treatments (e.g., filtration, desalination) for water quality improvement.

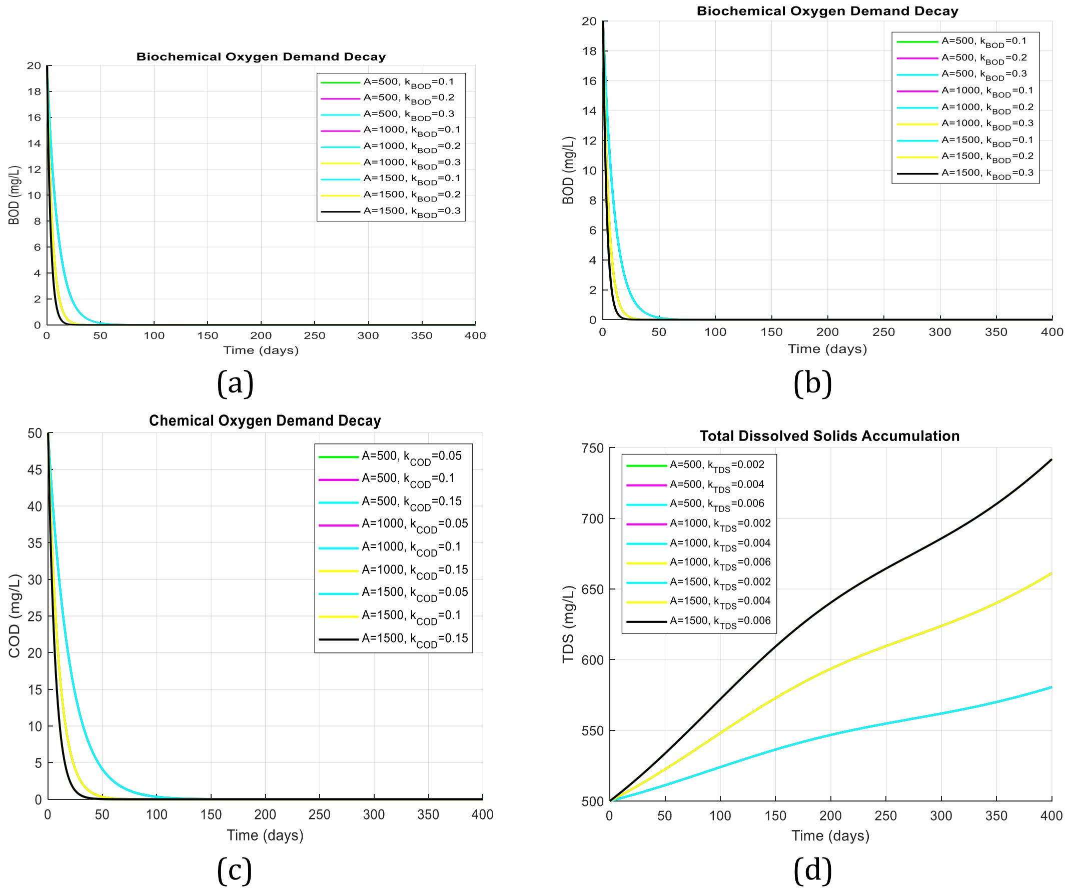

Figure 5: Graphs Showing the Effect of Varying Parameters on BOD, COD and TDS

Result and Discussion

The results of the numerical simulation illustrate the dynamic behaviour of Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), and Total Dissolved Solids (TDS) over 400 days under varying initial pollution loads and decay rates. The key observations are as follows:

Deterioration of Biochemical Oxygen Demand (BOD)

The exponential decline of Biochemical Oxygen Demand (BOD) over time parallels the natural decomposition of organic materials by microbes. Several significant factors influence the rate of this decline. Firstly, the decrease in BOD levels is directly proportional to the decay rate $K_d BOD$. This is expected, as a higher degradation rate suggests that contaminants are being more effectively broken down by microbes. Secondly, a higher initial pollution load (A) results in greater initial BOD concentration. Ultimately, every instance settles at a small steady-state number, despite some initial variability. Observations suggest that the BOD concentration stabilises near zero after approximately 100 days, indicating that most organic pollutants have decomposed. This trend aligns with the predictions of theoretical models of water quality, which indicate that microbial activity drives the BOD decay process. Chemical Oxygen Demand (COD) Decay: Similar to Biochemical Oxygen Demand (BOD), COD exhibits a slightly slower rate of decay, following an exponential decline. Several important elements influence this pattern. Firstly, a more rapid decrease in COD levels is associated with higher values of K_dCOD, indicating more effective oxidation of pollutants. Since COD encompasses both biodegradable and non-biodegradable organic matter, its concentrations ultimately exceed those of BOD. An increased initial pollution load (A) also results in a higher starting concentration of COD. Regardless of the pollution quantity, the general degradation trend remains consistent. It is expected that COD decomposes more slowly than BOD as it accounts for the total oxidation requirement of organic and inorganic contaminants. Moreover, due to the prolonged presence of non-biodegradable substances in the system, COD levels tend to stabilise at higher concentrations. Accumulation of Total Dissolved Solids (TDS): Total Dissolved Solids (TDS) rise steadily over time, in contrast to Biochemical Oxygen Demand (BOD) and Chemical Oxygen Demand (COD), which fall. Many things are pushing this tendency. To begin, flow dynamics (Q(t)) affect the buildup of dissolved solids, with small variations caused by seasonal variations. A more rapid buildup of dissolved chemicals is indicated by a steeper increase in TDS, which is caused by higher values of K_dTDS. There is no biological decomposition of TDS, in contrast to BOD and COD. On the contrary, dissolved solids remain in the water and progressively build up over time. This brings to light a major problem with water quality management: whereas organic contaminants can be broken down biologically, total dissolved solids (TDS) need specific treatment techniques like filtration or reverse osmosis to be effectively removed. The degradation of pollutants is accelerated, and the improvement of water quality is expedited when K_dBOD and K_dCOD are increased. Decay rates are unaffected by increased initial pollution loads (A), which lead to higher starting concentrations. Long-term management techniques that go beyond natural deterioration are necessary because TDS behaves differently than BOD and COD.

Conclusion

The study provides a thorough analysis of the changes that occur in three important water quality metrics over time and under different environmental conditions: Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), and Total Dissolved Solids (TDS). The data show that both BOD and COD have an exponential decay pattern, which means that organic contaminants are being broken down effectively by natural microbes. The delicate ecological balance of water bodies is sustained by this self-purification process. On the other hand, TDS accumulates gradually over time, which is a radically different tendency. Dissolved solids, in contrast to organic contaminants, do not break down biologically; thus, they remain in the system and worsen water quality over time. Increased water salinity, possible harm to aquatic life, and diminished appropriateness for industrial and domestic usage are some of the environmental problems posed by this. The importance of implementing efficient ways to control pollution is highlighted by these findings. First, improve the efficacy of wastewater treatment to hasten the removal of biochemical oxygen demand (BOD) and chemical oxygen demand (COD) to keep water quality from declining, remove as few non-biodegradable pollutants as possible from industrial effluent and to control the buildup of TDS, use cuttingedge treatment methods like filtration and reverse osmosis. In summary, the findings of this study highlight the need for coordinated approaches to water management if we are to leave our water supplies to the generations to come.

References

Barabas, S. (1975). *Water Quality Parameters*. ASTM International.

Ejigu, M. T. (2021). Overview of water quality modelling. *Cogent Engineering*, *8*(1), 1891711.

Darji, J., Lodha, P., & Tyagi, S. (2022). Assimilative capacity and water quality modelling of rivers: A review. *AQUA—Water Infrastructure, Ecosystems and Society*, *71*(10), 1127-1147.

D'Agostino, G. (2024). *Uncertainty Analysis of a River Quality Model Using Differential Inequalities* (Doctoral dissertation, University of Guelph).

Siwiec, T., Kiedrynska, L., Abramowicz, K., Rewicka, A. L. E. K. S. A. N. D. R. A., & Nowak, P. (2011). BOD measuring and modelling methods-review. *Annals of Warsaw University of Life Sciences-SGGW. Land Reclamation*, *43*(2).

Braun, H. B., & Berthouex, P. M. (1970). Analysis of lag phase BOD curves using the Monod equations. *Water Resources Research*, *6*(3), 838-844.

Gotovtsev, A. V. (2014). A new method for estimating BOD and the rate of biochemical oxidation based on modified Streeter-Phelp's equations. *Water Resources*, *41*, 330-334.

Shastry, J. S., Fan, L. T., & Erickson, L. E. (1973). Nonlinear parameter estimation in water quality modeling. *Journal of the Environmental Engineering Division*, *99*(3), 315-331.

Gotovstev, A. Determination of biochemical oxygen demand and the oxidation rate based on a modified Streeter-Phelps system. Doklady Earth Sciences 460, 2 (2015), 195–197.

Gotovtsev, A. V. (2010). Modification of the Streeter-Phelps system with the aim to account for the feedback between dissolved oxygen concentration and organic matter oxidation rate. *Water resources*, *37*, 245-251.

Saleh, A., Ibrahim, F. N., and Sharaf, M. Mathematical model for expanding suitable areas for fish survival in a polluted river by using the solution of the two coupled pollution and aeration equations. Modeling Earth Systems and Environment 10, 2 (2024), 1803–1813.

Masters, G. Introduction to Environmental Engineering and Science. Prentice-Hall, 1997.

Streeter, W., and Phelps, E. A study of the pollution and natural purification of the Ohio River. Tech. rep., United States Public Health Service, 1925.

Jia, L., & Stone, M. C. (2013, December). A framework of parameter uncertainty analysis based on hydro-catchments: Application of the VIC model to the Gila River basin. In *AGU Fall Meeting Abstracts* (Vol. 2013, pp. H33B-1348).

Wang, P. (2011). *Uncertainty quantification in environmental flow and transport models*. University of California, San Diego.

Zhang, Y. (2012). *Predicting river aquatic productivity and dissolved oxygen before and after dam removal in central Ohio, USA* (Master's thesis, The Ohio State University).

Olowe, K. O. (2018). Assessment of Some Existing Water Quality Models. *Nature Environment & Pollution Technology*, *17*(3).

Verma, R. K. (2010). Surface Water Quality Modelling and Minimum Flow Required for River System Management. *Computational Science*, 50.

Shraideh, H. A. (2014). Water Quality Modeling of Al-Qilt Stream. *Master of Water and Environmental Engineering, An Najah University*.

Ibrahim, T. N. B. T. (2021). *Application of Material Flow Analysis and Water Quality Modelling to Facilitate the Efficiency of Leachate Management System* (Doctoral dissertation, University of Malaya (Malaysia)).

Renjith, V. R. (2016). *Waste assimilative capacity studies of select regions of the west coast of India* (Doctoral dissertation, Goa University).

Baharudin, F., Kassim, J., Imran, S.N.M., Wahab, M.A., 2021. Water quality index (WQI) classification of rivers in agriculture and aquaculture catchments. IOP Conf. Ser. Earth Environ. Sci. 646 (1), 012023. https://doi.org/10.1088/1755-1315/646/1/012023.

Abidin, M.Z., Kutty, A.A., Lihan, T., Zakaria, N.A., 2018. Hydrological change effects on Sungai Langat water quality. Sains Malaysiana 47, 1401–1411. https://doi.org/10.17576/jsm-2018-4707-07.

AbuHasan, H., Abdullah,S., Kamarudin, S.K., Kofli,N., 2011. Ammonia and manganese problems in Malaysian drinking water treatment. World Appl. Sci. J. 12, 1890–1896.

Ahmed, M., Mokhtar, M., Alam, L., Choo Ta, G., Lee, K.E., Md Khalid, R., 2018. Recognition of local authority for better management of drinking water at the Langat River Basin, Malaysia. Int. J. Eng. Technol. 7, 148–154. https://doi.org/10.14419/ijet.v7i3.30.18217.

Air Selangor, 2020. Unscheduled water supply disruption in 7 regions starting at 6.00 PM today following raw water pollution. Retrieved 21 January 2021, from https://www.airselangor.com/gangguan-bekalan-air-tidak-berjadual-di-7-wilayah-bermula-jam-600 petang-hari-ini-berikutan-pencemaran-sumber-air-mentah/.

Air Selangor, 2021. Introducing Air Selangor's new water treatment plant- Rasau Water Supply Scheme. Retrieved 3 June 2021 from https://www.airselangor.com/introducing-air
selangors-new-water-treatment-plant-rasau-water-supply-scheme/.

Al-Badaii, F., Shuhaimi-Othman, M., Gasim, M.B., 2013. Water quality assessment of the Semenyih River, Selangor, Malaysia. J. Chem. 2013, 871056. https://doi.org/10.1155/2013/871056.

Ali, R., Kuriqi, A., Abubaker, S., Kisi, O., 2019. Long-term trends and seasonality detection of the observed flow in the Yangtze River using Mann-Kendall and Sen's innovative trend method. Water 11 (9), 1855. https://doi.org/10.3390/w11091855.

Arslan, H., 2013. Application of multivariate statistical techniques in the assessment of groundwater quality in seawater intrusion area in Bafra Plain, Turkey. Environ. Monit. Assess. 185 (3), 2439–2452. https://doi.org/10.1007/s10661-012-2722-x.

Azim, I., 2020. The fault in our waters. Retrieved from http://thehubbmovement.org/the fault-in-our-waters.html.

Baharudin, F., Kassim, J., Imran, S.N.M., Wahab, M.A., 2021. Water quality index (WQI) classification of rivers in agriculture and aquaculture catchments. IOP Conf. Ser. Earth Environ. Sci. 646 (1), 012023. https://doi.org/10.1088/1755-1315/646/1/012023.

Jesi, A.A., Feisal, N.A.S., Windusari, Y., Samat, N.A., Kamaludin, N.H., Derahim, N., Ahmad, M.A., Azhar, T.N.T., & Ibrahim, T.N.B.T. (2024). Klang River Water Quality Assessment and Its Effects on Human Health Using Chemometric Analysis. Journal of Environmental Health. 16(2).

Loi, J. X., Chua, A. S. M., Rabuni, M. F., Tan, C. K., Lai, S. H., Takemura, Y., & Syutsubo, K. (2022). Water quality assessment and pollution threat to safe water supply for three river basins in Malaysia. *Science of The Total Environment*, *832*, 155067.

Naji, A., & Ismail, A. (2012). Metals Fractionation and Evaluation of their Risk Connected with Urban and Industrial Influx in the Klang River Surface Sediments, Malaysia. *EnvironmentAsia*, *5*(1). https://www.thaiscience.info/journals/Article/ENVA/10812032.pdf